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# Sirindhorn International Institute of Technology Thammasat University 

Midterm Examination: Semester 1 / 2018

Course Title: ECS332 (Principles of Communications)
Instructor: Asst. Prof. Dr.Prapun Suksompong
Date/Time: October 5, 2018 / 09:00-11:00

## Instructions:

> This examination has.......... pages (including this cover page).
> Conditions of Examination:

## $\square$ Open book

Closed book
$\boxed{\square}$ Semi-Closed book (.................sheet(s) $\checkmark 1$ page $\square$ both sides of A4 paper note)
This sheet must be hand-written. They should be submitted with the exam.
Do not modify (,e.g., add/underline/highlight) content on the sheet inside the exam room.
Indicate your name and ID in the upper-right corner of the sheet (in portrait orientation).
Other requirements are specified on the course website. ( -10 pt if not following the requirements.)
$\square$ other:
$\square$ No dictionary $\square$ Dictionary allowed $\square$ No calculator $\square$ Calculator allowed
$>$ Read these instructions and the questions carefully.
$>$ Students are not allowed to be out of the examination room during examination. Going to the restroom may result in score deduction.
$>$ Turn off all communication devices and place them with other personal belongings in the area designated by the proctors or outside the test room.
> Write your name, student ID, section, and seat number clearly in the spaces provided on the top of this sheet. Then, write your first name and the last three digits of your ID in the spaces provided on the top of each page of your examination paper, starting from page 2 .
$>$ The examination paper is not allowed to be taken out of the examination room. Violation will result in a zero (0) score for the examination. Also, do not remove the staple.
$>$ Unless instructed otherwise, write down all the steps that you have done to obtain your answers.

- When applying formula(s), state clearly which formula(s) you are applying before plugging-in numerical values.
- You may not get any credit even when your final answer is correct without showing how you get your answer.
- Formula(s) not discussed in class can be used. However, derivation must also be provided.
- Exceptions:
- Problems that are labeled with "ENRPr" (Explanation is not required for this problem.)
- Parts that are labeled with "ENRPa" (Explanation is not required for this part.)
- These problems/parts are graded solely on your answers. There is no partial credit and it is not necessary to write down your explanation. Usually, spaces (boxes or cells in a table or rows of dashes) will be provided for your answers. "WACSP" stands for "write your answer(s) in the corresponding space(s) provided".
$>$ The back of each page will not be graded; it can be used for calculations of problems that do not require explanation.
> When not explicitly stated/defined, all notations and definitions follow ones given in lecture. For example, the sinc function is defined by $\operatorname{sinc}(x)=(\sin x) / x$; time is denoted by $t$ and frequency is denoted by $f$. The unit of $t$ is in seconds and the unit of $f$ is in Hz .
$>$ Some points are reserved for accuracy of the answers and also for reducing answers into their simplest forms. Watch out for roundoff error. Unless specified otherwise, the error in your final answer should not exceed $0.1 \%$.
$>$ Points marked with * indicate challenging problems.
$>$ Do not cheat. Do not panic. Allocate your time wisely.
$>$ Don't forget to submit your fist online self-evaluation form by the end of today.
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1. $(1 \times 2+0.5 \times 7=5.5 \mathrm{pt})[\mathrm{ENRPr}]$ Signals $x(t), y(t)$, and $z(t)$ are plotted below.




Suppose $y(t)=c_{1} x\left(c_{2} t+c_{3}\right)$ and $z(t)=c_{4} x\left(c_{5} t+c_{6}\right)+c_{7} x\left(c_{8} t+c_{9}\right)$.
Find the values of the constants $c_{1}, c_{2}, \ldots, c_{9}$

$$
\begin{array}{lll}
c_{1}= & , & c_{2}= \\
c_{4}= & c_{3}= \\
c_{7}= & c_{5}= & c_{6}= \\
, & c_{8}= & c_{9}=
\end{array}
$$

2. $(8 \mathrm{pt})$ [ENRPr] Consider two signals $m(t)$ and $g(t)$.

Their Fourier transforms are shown below. Note that $M(f)=c_{1} \operatorname{sinc}\left(c_{2} f\right)$.


In the time domain, suppose $g(t)=c_{3} m(t) \cos \left(c_{4} t\right)$.
(a) $(3 \mathrm{pt})$ Plot $m(t)$

(b) (4 pt) Find the values of the constants $c_{1}, c_{2}, \ldots, c_{4}$ : $c_{1}=$ $\qquad$ , $c_{2}=$ $\qquad$ , $c_{3}=$ $\qquad$ , $c_{4}=$ $\qquad$ .
(c) (1 pt) Find $\int_{-\infty}^{\infty} G(f) d f$.
3. $\left(3+0.5^{* *}=3.5 \mathrm{pt}\right)$ [ENRPr] Evaluate the following integrals:
a. $\int_{-\infty}^{\infty} 2 \delta(t-2) d t=$ $\qquad$ b. $\int_{-\infty}^{\infty} \delta(3 t) d t=$
c. $\int_{1}^{7} \delta(t-3) \cos \left(\frac{\pi}{2} t\right) d t=$ $\qquad$ d. $\int_{0}^{1} \delta\left(e^{2 t}-2\right) d t=$ $\qquad$
$\qquad$
4. $(7 \mathrm{pt})$ [ENRPr] The impulse response of a multipath channel is of the form

$$
h(t)=\sum_{k=1}^{v} \beta_{k} \delta\left(t-\tau_{k}\right) .
$$

a. (4 pt) Plot $|H(f)|$ for the parameters given in each part below.

Consider the frequency from $f=-1$ to $f=1 \mathrm{~Hz}$.
i. (2 pt) $v=1, \beta_{1}=0.5, \tau_{1}=3$

ii. (1 pt) $v=2, \beta_{1}=\beta_{2}=0.5, \tau_{1}=1, \tau_{2}=3$

iii. $(1 * \mathrm{pt}) v=3, \beta_{1}=\beta_{3}=0.5, \beta_{2}=1, \tau_{1}=1, \tau_{2}=2, \tau_{3}=3$

b. (3 pt) Suppose $v=2, \beta_{1}=\beta_{2}=0.5, \tau_{1}=1, \tau_{2}=3$. For each of the following channel input $x(t)$, find the corresponding channel output $y(t)$.
Note that the output should be of the form $y(t)=A \cos \left(2 \pi f_{c} t+\theta\right)$ for some constants $A, f_{c}$, and $\theta$.

| Channel input | Channel output |
| :--- | :--- |
| $x(t)=\cos (\pi t)$ |  |
| $x(t)=\cos \left(\frac{\pi}{2} t\right)$ |  |
| $x(t)=\cos \left(\frac{\pi}{4} t\right)$ |  |

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5. (5 pt) [ENRPr] Consider an LTI communication channel.

Suppose when we put

$$
x(t)=4 \cos (\pi t)+2 \cos (2 \pi t)+\cos (4 \pi t)+0.5 \cos (6 \pi t)+\cos (8 \pi t)+1
$$

into this channel, we get

$$
y(t)=0.5 \cos (\pi t)+e^{j 2 \pi t}+2 \sin (4 \pi t)+4 \cos (6 \pi t)+8
$$

as its output.
Let $H(f)$ be the frequency response of the channel that satisfies the above input-output relation. Find $H(f)$ at $f=-3,-1,1,2,3$.

| $f$ | -3 | -1 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $H(f)$ |  |  |  |  |  |

6. (6 pt) [ENRPr] Consider the "modulator" shown below. Note that the first operation is a summation (not multiplication).

$(\cdot)^{2}$ is a "square" device; its output is created by squaring its input in the time domain. $H_{\mathrm{BP}}(f)$ is an LTI device whose frequency response is

$$
H_{\mathrm{BP}}(f)= \begin{cases}1, & \left|f-f_{c}\right| \leq 332 \\ 1, & \left|f+f_{c}\right| \leq 332 \\ 0, & \text { otherwise }\end{cases}
$$

Let $A_{c}=2$ and $f_{c}=2018$. In each part below, plot the corresponding $X(f)$.

|  |  |  |
| :--- | :--- | :--- |
| a. $m(t)=\cos (400 \pi t)$. |  | $X(f)$ |
|  |  |  |
| b. $M(f)= \begin{cases}1, & \|f\| \leq 54, \\ 0, & \text { otherwise. }\end{cases}$ |  |  |

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7. $(8 \mathrm{pt})$ [ENRPr] Consider the DSB-SC modem with no channel impairment shown below.


The Fourier transform of the message is plotted below.




Let $A_{1}=2, A_{2}=4$, and $f_{\mathrm{c}}=40 \mathrm{~Hz}$.
a. $\quad(3+4=7 \mathrm{pt})$ Plot $X(f)$ and $V(f)$ in the provided space above.
b. (1 pt) Suppose the low-pass filter (LPF) is ideal with frequency response

$$
H_{L P}(f)= \begin{cases}g, & |f| \leq 30 \\ 0, & \text { otherwise }\end{cases}
$$

Find the value of $g$ that makes $\hat{m}(t)=m(t)$.

$$
g=
$$

$\qquad$ .
8. (6 pt) [ENRPr] Consider a DSB-SC modem with no channel impairment shown below.


Let $A_{1}=2, A_{2}=4$, and $f_{\mathrm{c}}=2018 \mathrm{~Hz}$. Suppose LPF has $H_{L P}(f)= \begin{cases}1, & |f| \leq 332, \\ 0, & \text { otherwise } .\end{cases}$
For each of the following $m(t)$, find the corresponding $\hat{m}(t)$.

| $m(t)$ | $\hat{m}(t)$ |
| :--- | :--- |
| $m(t)=4 \cos (444 \pi t)$ |  |
| $m(t)=8 \cos (7654 \pi t)$ |  |
| $m(t)=10 \cos (8420 \pi t)$ |  |

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9. (2 pt) Consider a DSB-SC modem with synchronization error shown below.


Let $A_{1}=2, A_{2}=4, f_{\mathrm{c}}=2018 \mathrm{~Hz}$, and $H_{L P}(f)= \begin{cases}1, & |f| \leq 332, \\ 0, & \text { otherwise } .\end{cases}$
Suppose $\theta_{1}=\frac{\pi}{2}$ [rad], $\theta_{2}=-\frac{\pi}{2}$ [rad], and $\Delta f=1 \mathrm{~Hz}$.
Assume that $m(t)$ is band-limited to 100 Hz . Find $\hat{m}(t)$.
(Because $m(t)$ is not specified, your answer should be expressed in terms of $m(t)$.)
10. (9 pt) [ENRPr] For each of the following signal $g(t)$, find its (normalized) average power $\left.\left.P_{g} \equiv\langle | g(t)\right|^{2}\right\rangle$. Do not use any approximation.

| $g(t)$ | $\left.\left.P_{g} \equiv\langle \| g(t)\right\|^{2}\right\rangle$ |
| :--- | :--- |
| $(1 \mathrm{pt}) g(t)=10 e^{j 20 \pi t}$ |  |
| $(1 \mathrm{pt}) g(t)=10 e^{j 20 \pi t}+5 e^{j 40 \pi t}$ |  |
| $(1 \mathrm{pt}) g(t)=\left(10 e^{j 20 \pi t}+5 e^{j 40 \pi t}\right)^{2}$ |  |
| $(2 \mathrm{pt}) g(t)=4 \cos \left(4 t+4^{\circ}\right)$ |  |
| $(2 \mathrm{pt}) g(t)=5 \cos \left(3 t+15^{\circ}\right)+12 \cos \left(4 t+105^{\circ}\right)$ |  |
| $(2 \mathrm{pt}) g(t)=5 \cos \left(3 t+15^{\circ}\right)+12 \cos \left(3 t+105^{\circ}\right)$ |  |

11. (6 pt) Consider a signal $g(t)$ below. Note that $g(t)=0$ outside of the interval [0,2].


Let $y(t)=\sum_{k=-\infty}^{\infty} g(t-k)$ and $z(t)=\sum_{k=-\infty}^{\infty} g(t-2 k)$.
Calculate the following quantities:
a. $(2 \mathrm{pt})$ energy $E_{g}$
b. $(1 \mathrm{pt})$ average power $P_{g}$
c. $(1 \mathrm{pt})\langle g(t)\rangle$
d. $\left(1^{*} \mathrm{pt}\right)[\mathrm{ENRPa}]$ average power $P_{y}$
e. (1* pt) [ENRPa] average power $P_{z}$
12. ( 1 pt )
a. (1 pt) Do not forget to submit your study sheet with your exam.
b. Reminder:
i. Make sure that you write your name and ID on every page. (Read the instruction on the cover page.)
ii. The online self-evaluation form is due by the end of today.

